NASA TECHNICAL TRANSLATION NASA TT F-12,965

LAMINAR BOUNDARY LAYER IN A COMPRESSED GAS AT A GIVEN SURFACE TEMPERATURE AND IN THE PRESENCE OF SUCTION

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Translation of "Laminarnyy Pogranichnyy sloy v Szhimayemom Gaze Pri Zadannoy Temperature Poverkhnosti i Nalichii Otsasyvaniya," Prikladnaya Mekhanika, Vol. 5, No. 7, 1969, pp. 93-98.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 APRIL 1970

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LAMINAR BOUNDARY LAYER IN A COMPRESSED GAS AT A GIVEN SURFACE TEMPERATURE AND IN THE PRESENCE OF SUCTION

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ABSTRACT. An approximate method for calculating the characteristics of the laminar boundary layer in a compressed gas at a given surface temperature when suction of the gas through the permeable surface of the body is present, is presented. The proposed method is applicable to a two-dimensional flow, but the same equation can be used in the case of axially symmetrical boundary layers on a body of revolution.

Reference [4] proposed a method for making an approximate calculation of the characteristics of the laminar boundary layer on a thermally insulating surface at high gas velocities and in the presence of suction. The method is based on the use of the integral pulse relationship in A. A. Dorodnitsyn's variables and of an approximation of a family of velocity profiles perpendicular to the boundary layer by sixth degree polynomials. Calculation can also be based on a system of integral "three moments" relationships. This article develops an approximate method for calculating the characteristics of the laminar boundary layer in a compressed gas at a given surface temperature and when suction of the gas through the permeable surface of the body is present. /93*

Let us consider the high-velocity flow of a stationary stream of compressed gas over a plane airfoil profile. We can introduce a system of coordinates, the origin of which is located at the forward critical point. The x axis is directed along the surface, and the y axis is directed along the normal to the surface of the profile. Setting the Prandtl number equal to unity, the system of differential equations for the laminar boundary layer can be written in the following form, taking into account the longitudinal

* Numbers in the margin indicate pagination in the foreign text.

pressure gradient on the outer boundaries:

Prandtl equation

$$\varrho u \frac{\partial u}{\partial x} + \varrho v \frac{\partial u}{\partial y} = -\frac{dp_\delta}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right); \tag{1}$$

continuity equation

$$\frac{\partial}{\partial x}(\varrho u) + \frac{\partial}{\partial y}(\varrho v) = 0; \tag{2}$$

energy balance equation (approximate)

$$\varrho u \frac{\partial \theta}{\partial x} + \varrho v \frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial \theta}{\partial y}\right); \tag{3}$$

Clapeyron equation

$$p_\delta = \varrho RT; \tag{4}$$

exponential dependence

$$\mu = \mu_1 \left(\frac{T}{T_\delta}\right)^n. \tag{5}$$

Here u and v are projections of the velocity vector in the boundary layer on the coordinate axis; p_δ (x) is the preassigned pressure distribution along the outer boundary layer; $\theta = T + u^2/2JC_p$ is the drag temperature; T is the absolute temperature; J is the mechanical equivalent of heat; $R = J\,(C_p - C_v)$ is the universal gas constant; C_p and C_v are the heat capacities of gas at constant pressure and constant volume respectively; μ and ϱ are the dynamic viscosity coefficient and the density of the gas respectively. /94

The values of the same quantities under the adiabatically and isentropically retarded flow will be designated by index 1, and the values of these quantities at the outer limit of the boundary layer will be designated by the index δ.

For air we can take R = 287.1 $m^2/deg.sec^2$ and n = 0.75 at $-23° \leq T \leq 327°C$.

The system of equations (1 - 5) is a closed system, and therefore may be used for the determination of the following unknown quantities: two velocity components, pressure, density and the temperature of the gas.

The following boundary conditions are satisfied on the permeable surface of the profile in the presence of suction, and at the outer limit of the boundary layer.

$$u = 0; \quad v = v_0; \quad T = T_0 \text{ when } y = 0; \tag{6}$$

$$u = U_\delta; \quad T = T_\delta \text{ when } y = \delta, \tag{7}$$

where v_0 (x) is the local rate of suction; T_0 is the surface temperature; U_δ and T_δ are the velocity and temperature of the gas at the outer boundary of the layer, respectively; δ is the thickness of the boundary layer.

In the case considered, where heat exchange through the surface of the profile takes place, the integral of the energy balance equation at (3) is equal to [2]

$$\theta = C_1 u + C_2. \tag{8}$$

Utilizing the boundary conditions at (6) and (7) to determine the values of constants of integration C_1 and C_2 we obtain

$$\frac{T}{\theta_0} = \left[1 - \bar{u}^2 + \left(\frac{T_0}{\theta_0} - 1\right)\left(1 - \frac{\bar{u}}{\bar{U}}\right)\right]. \tag{9}$$

Here $\theta_0 = T_\delta + \frac{U_\delta^2}{2JC_n}$ is the drag temperature of the oncoming stream;

$\bar{u} = u/\sqrt{2i_0}$; $\bar{U} = U_\delta/\sqrt{2i_0}$; $i_0 = JC_pT_\delta + \frac{U_\delta^2}{2}$ is the total energy.

The pressure can be calculated through the Bernoulli equation

$$p = p_{\delta_1}\left(1 - \frac{U_\delta^2}{2i_0}\right)^{\varkappa^*} \quad \left(\varkappa^* = \frac{\varkappa}{\varkappa - 1}\right), \tag{10}$$

and the Clapeyron equation at (4) can be used to determine an equation for density

$$\varrho = \varrho_{\delta_1}\frac{(1 - \bar{U}^2)^{\varkappa^*}}{1 - \bar{u}^2 + (T_0/\theta_0 - 1)(1 - \bar{u}/\bar{U})}, \tag{11}$$

where the ratio of the heat capacity coefficients is equal to $\varkappa = C_p/C_v$.

Using the exponential relationship at (5), we find

$$\mu=\mu_1\left[1-\bar{u}^2+\left(\frac{T_0}{\theta_0}-1\right)\left(1-\frac{\bar{u}}{\bar{U}}\right)\right]^n. \tag{12}$$

Further calculations will be made using the A. A. Dorodnitsyn's /95
variables [1]. In this case these are in form

$$\xi=\int_0^x(1-\bar{U}^2)^{\varkappa^*}dx;\quad \eta=\int_0^y\frac{(1-\bar{U}^2)^{\varkappa^*}}{1-\bar{u}^2+(T_0/\theta_0-1)(1-\bar{u}/\bar{U})}\,dy. \tag{13}$$

The formulas transformed to the new variables will appear as follows:

$$\frac{\partial}{\partial x}=(1-\bar{U}^2)^{\varkappa^*}\frac{\partial}{\partial\xi}+\frac{\partial\eta}{\partial x}\frac{\partial}{\partial\eta}; \tag{14}$$

$$\frac{\partial}{\partial y}=\frac{(1-\bar{U}^2)^{\varkappa^*}}{1-\bar{u}^2+(T_0/\theta_0-1)(1-\bar{u}/\bar{U})}\frac{\partial}{\partial\eta}. \tag{15}$$

Utilizing the equations at (13 - 15), the boundary layer equation at (1), and the continuity equation at (2) can be reduced to the following form

$$u\frac{\partial u}{\partial\xi}+w\frac{\partial u}{\partial\eta}=\frac{1-\bar{u}^2+(T_0/\theta_0-1)\left(1-\frac{\bar{u}}{\bar{U}}\right)}{1-\bar{U}^2}U_\delta U_\delta'+$$

$$+\nu_{\delta_1}\frac{\partial}{\partial\eta}\left\{\left[1-\bar{u}^2+(T_0/\theta_0-1)\left(1-\frac{\bar{u}}{\bar{U}}\right)\right]^{n-1}\frac{\partial\bar{u}}{\partial\eta}\right\}; \tag{16}$$

$$\frac{\partial u}{\partial\xi}+\frac{\partial w}{\partial\eta}=0, \tag{17}$$

where

$$w=\frac{u}{\left[1-\bar{u}^2+(T_0/\theta_0-1)\left(1-\frac{\bar{u}}{\bar{U}}\right)\right]}+\frac{\bar{u}}{(1-\bar{U}^2)^{\varkappa^*}}\frac{d\eta}{dx};$$

$$U_\delta'=dU_\delta/d\xi.$$

After the transformation of the equation at (16) by that at (17), and term by term integration with respect to η from 0 to δ_η, we obtain

$$\frac{d}{d\xi}\int_0^{\delta_\eta}u(U_\delta-u)\,d\eta+\int_0^{\delta_\eta}\frac{\partial}{\partial\eta}[w(U_\delta-u)]\quad d\eta-$$

$$-U_\delta' \int_0^{\delta_\eta} \left\{ \frac{\left[(1-\bar{u}^2+(T_0/\theta_0-1)\left(1-\frac{\bar{u}}{\bar{U}}\right)\right]}{1-\bar{U}^2} U_\delta - \bar{u} \right\} d\eta = \nu_{\delta_1} \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}, \tag{18}$$

where δ_η is the thickness of the boundary layer along the η coordinate.

Since $\bar{u} = 0$ and $\eta = 0$ the integral has the following form

$$\int_0^{\delta_\eta} \frac{\partial}{\partial \eta} [w(U_\delta - u)]\, d\eta \doteq U_\delta v_0.$$

Integrating the equation at (18), and taking the expression at (15) into consideration we can write: /96

$$\frac{df}{d\xi} - f \frac{d}{d\xi} \ln \frac{U_\delta'}{(1-\bar{U}^2)^2} = [F(f) - t^{**}] \frac{d}{d\xi} \ln \frac{U_\delta}{\sqrt{1-\bar{U}^2}}. \tag{19}$$

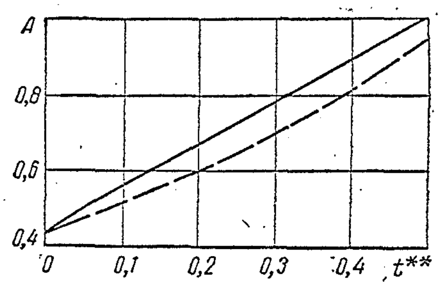

Figure 1.

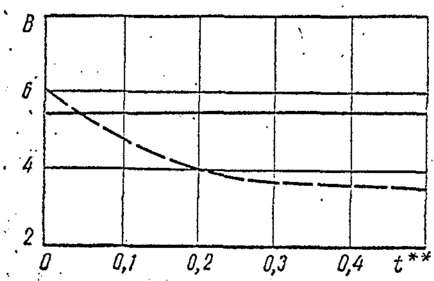

Figure 2.

Here we introduce the following designations:

$$\delta_\eta^* = \int_0^{\delta_\eta} \left(1 - \frac{u}{U_\delta}\right) d\eta; \quad \delta_\eta^{**} = \int_0^{\delta_\eta} \frac{u}{U_\delta}\left(1 - \frac{u}{U_\delta}\right) d\eta; \tag{20}$$

$$f = \frac{U_\delta' \delta_\eta^{**2}}{\nu_{\delta_1}(1-\bar{U}^2)}; \quad t^{**} = \frac{v_0 \delta_\eta^{**}}{\nu_{\delta_1}}; \tag{21}$$

$$\xi(f, t^{**}) = \frac{\delta_\eta^{**}}{U_\delta}\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}; \quad H(f, t^{**}) = \frac{\delta_\eta^*}{\delta_\eta^{**}}; \tag{22}$$

$$F(f, t^{**}) = 2\{\xi(f, t^{**}) - [2 + H(f, t^{**})] f\}. \tag{23}$$

Returning to the physical plane x, y, the integral equation at (19) can be reduced to the final form

$$\frac{df}{dx}-f\frac{d}{dx}\ln\frac{dU_\delta/dx}{(1-\bar{U}^2)^{2+\varkappa^*}}=[F(f,t^{**})-t^{**}]\frac{d}{dx}\frac{U_\delta}{\sqrt{1-\bar{U}^2}}. \quad (24)$$

Let us assume that the velocity profiles transverse to the boundary layer, constitute a monoparametric family and are independent in the explicit form of the Mach number. Such an assumption reflects the actual velocity distribution across the laminar boundary layer when the Mach numbers are less than 2 [6], quite satisfactorily. The suction of the gas from the boundary layer through the permeable surface of the profiles can be represented by the function F (f, t**) in the following form [3, 5]:

$$F(f,t^{**})=A(t^{**})-B(t^{**})f. \quad (25)$$

The numerical values of A and B as a function of the suction parameter t** are given in Figures 1 and 2. In these graphs, as in Figures 3 and 4, solid lines represent the data obtained in [3], and the broken lines represent the corresponding data cited in reference [5].

Integrating the differential equation at (24), and taking into account the linear dependence of (25), we find

$$f(x)=\frac{dU_\delta/dx}{U_\delta^B(1-\bar{U}^2)^N}\int_0^x U_\delta^{B-1}(A-2t^{**})(1-\bar{U}^2)^{N-1}dx, \quad (26)$$

where

$$N=2+\varkappa^*-\frac{B}{2}. \quad (26a)$$ /97

The constant integration of the expression at (26) is equal to zero since the form parameter f is finite when x = 0, because the system of coordinates was selected so that U = 0 when x = 0. In this specific case we have f (0) = A/B. The values of the form parameter at the forward critical point f (0) are represented in Figure 3, and the quantity N is represented in Figure 4.

The successive approximations method must be used to calculate the form parameter f (x) through the formula at (26) for a given normal velocity component on the surface, v_0. As the first approximation one can calculate the values of the form parameter f (x) by assigning the expected change in the suction parameter t** through the formula at (26). Then, using well-known formulas [3], one can obtain the second approximation of the value t**. Repeating this process of successive approximations one can calculate all of the characteristics of the boundary layer with sufficient accuracy.

As the first approximation of the expected change in the parameters t**, it is recommended that corresponding values for the porous plate (U_δ = 0) be assigned along with the assigned velocity distribution, v_0, for the airfoil profile with a porous surface.

The diagram shown on page 4 of monograph [6] is recommended for converting the known pressure distribution coefficient along the surface of the airfoil profile in the incompressible liquid to the U (x) distribution for different Mach numbers (lower than the critical value).

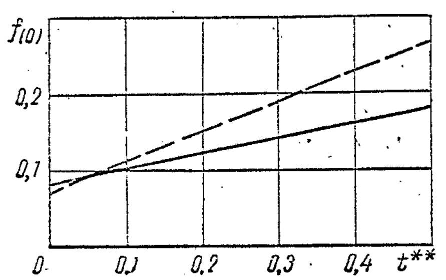

Figure 3.

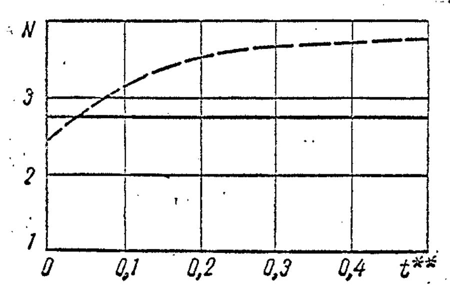

Figure 4.

Calculating the values of the form parameter f (x) through the relationship at (26), and utilizing the functions ζ (f, t**) and H (f, t**), cited in references [3,5], we can determine all the remaining characteristics of the laminar boundary layer in a compressed gas with present suction through the porous surface of the profile through formulas (21 - 23).

The proposed method is applicable to a two-dimensional flow. The same equation can be used in the case of axially symmetrical boundary layers on a body of revolution. Only the form of the continuity equation at (2) is changed, adding a cofactor, r_0, in the parentheses to represent the radius of the cross-section of the body of revolution. In this case the integral at (23) will become

$$f(x) = \frac{dU_\delta/dx}{U_\delta^B r_0^2 (1-\bar{U}^2)^N} \int_0^x U_\delta^{B-1} r_0^2 (A - 2t^{**})(1-\bar{U}^2)^{N-1} dx. \tag{27}$$

The other characteristics of the laminar boundary layer on the body of revolution can be calculated through the same formulas as for the two-dimensional case. /98

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Submitted for publication
30 September 1967

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Translated for the National Aeronautics and Space Administration under contract No. NASw-2038 by Translation Consultants, Ltd., 944 South Wakefield Street, Arlington, Virginia 22204.